A Case Study of the Role of Ethnomathematics among Teacher Education Students from Highly Diverse Cultural Backgrounds

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Abstract

Mathematics prospective teachers (MPT), Jewish and Bedouin alike, who came together in the Ethnomathematics Program (EP) in the Kaye College of Education (Israel), explored, identified, and learned naturally occurring mathematics in the cultures of their own people. They developed projects based upon their explorations and shared them in class. The MPT participated in collaborative discussions and formed intercultural understanding within a purposefully conducive classroom atmosphere. In particular, they shared with one another classroom activities for their classmates, plans for transferring their accomplishments to school children, ethnomathematics activities derived directly from cultural experiences, as well as mathematics content and pedagogy. This case study investigated and documented the intentional integration of two people’s cultures, traditions, and their ethnomathematics, on the one hand, with mathematics content, history, and pedagogy, on the other. The MPT learned ethnomathematics in the cultures of their own people and came to perceive mathematical concepts as a mix of two significances: one meaning stemming from their abstract courses in academic mathematics, a focus preeminent in western culture, and the second, stemming from comprehension of practical math concepts rooted for both Jews and Arabs in their cultures of the East. The following perception arose during class debates, discussions, and sharing of projects: that there is some common religious knowledge (e. g., that of the number seven in passages in the Torah and in the Koran) and common ritual (e. g., Jewish and Muslim religious women must cover their heads) in both lines of the descendants of Abraham. However, the big take-away point is that just an exposure to mathematics rooted in socio-cultural settings of their own people was not sufficient to have altered the standpoints of the class members toward the way in which mathematical concepts should be seen. Inquiry into situations where mathematical practice actually occurs was necessary.
Introduction

Globalization has brought about demographic changes and a plurality of cultures in various countries. The different aspects relating to multi-cultural society are becoming increasingly significant to education in general and mathematics education in particular. Discussing the face of mathematics from this point of view has the potential of understanding mathematics as an area whose face is diverse and can be interpreted in both formal and local aspects. The first interpretation as recognized in its formal definition, which can be conceptualized by the mathematician and is sufficiently absolute, perceptually speaking, is to be understood by learners thousands of miles apart – one, perhaps, in Australia and the other in Brazil or Africa. The second interpretation stems from expanding the world of the mathematical concepts by local aspects that are influenced by various explanations – culture, era and human history.

This realization requires us to hold a purposeful discussion about the added values in mathematics education. It is possible that Ethnomathematics, which is an expression of a Multicultural View of Mathematics Education and was launched as a new area at the 1985 NCTM by D'Ambrosio, helps us understand the role of mathematics in a multi-cultural society. The meaning of the term, Ethnomathematics, expressed by its socio-cultural aspect, includes within it the language, verbal lexicon, behavioral norms and symbols of a certain group, and in its mathematical aspect it includes such actions as "the corpus of knowledge derived from quantitative and qualitative practices, such as counting, weighing and measuring, sorting and classifying" (D’Ambrosio, 1985), encoding, sequencing, drawing a conclusion, exemplification, formation, location, explanation and reconstruction (Bishop, 1988). Regarding the actions that are identified with the mathematical aspect, there are a wide variety of existing implementations,
enabling explanation of the processes that characterize practical mathematics in traditional work. The reference is to the accumulated store of mathematical knowledge that over time becomes "the oral law that is passed from father to son and from mother to daughter" (Katsap, 2004). For instance, a Bedouin woman observing her rituals will instruct her daughters how to embroider, and put into practice compositions requiring basic comprehensive knowledge in the field of Geometry, such as squares, triangles, and symmetry. However, often the people involved in math education have been unaware of the actual fact that "mathematical procedures, patterns, and structures are developed by human beings, living and working in societies," or ignoring them altogether (Shirley, 2006).

**Ethnomathematics and Teachers Education: Foreword**

Professional literature, including the research literature, on the subject of multiculturalism in mathematics, responds at two levels: pure mathematics and mathematics education. On the first level, the following aspects are discussed: the nature of multiculturalism in mathematics, identification of cultural components in algorithms and in mathematical models, and the disclosure of differences in the presentation of mathematical objects in different cultures. On the second level, multiculturalism in mathematics is examined through the prism of mathematics education in all its components. The subjects discussed on this level include such topics as explanation by researchers of the points of view of those engaged in mathematics education and of the students in different cultures, regarding their concepts of algorithms or mathematical models, besides the training of mathematics teachers.

The researchers concluded that "there is a great necessity for ethnomathematics-based programs to identify and seek teaching-learning practices directed towards pedagogical action"(Orey & Rosa, 2005). Hence, the direction is to search for a way to design, through
collaboration with the teachers, a curriculum linked to the cultural needs of the learners (Masingila, 1996), to identify the preferable strategy of applying ethnomathematics in teaching (Bisher, 2001), and to use ethnomathematics "as an educational tool to help students understand what mathematics is about, and to help them make it part of their own knowledge" (Adam, 2004).

Shirley (2001), who conducted a survey of published projects, researches and essays, concludes that today the field of Ethnomathematics is central to the process of teaching and teaching methods, and that there is no reason to avoid its inclusion it in the curricula of mathematics teacher training. However, there is a gap between the ideal and the reality, and the current state of affairs is that in most cases mathematics departments of educational colleges are preparing teachers for mathematics discipline without any cultural context. This situation induces the future teachers to think of mathematics as universal and "acultural" (D'Ambrosio, 2001). Since this is the way things look at present, educators must think how to change the situation and help the teachers to establish cultural models in the classroom. This solution seems to be potential pedagogical guidance that takes into account the knowledge the students learn outside of the classroom. Likewise, educators need to broaden the understanding of mathematics prospective teachers (MPT) regarding the influence socio-cultural values have on the grasping of mathematical concepts on the one hand, and how mathematics is involved in the communal life-experience of different cultural groups, on the other.

In this article, we will focus on some characteristics of the second level.

The background of course population and EP conditions

The History of Mathematics seemed an appropriate course at Israel’s Kaye College of Education for incorporating the Ethnomathematical Program (EP). It provided opportunities for
aspiring teachers to examine the cultural contexts of mathematics concepts, to further their understanding regarding multicultural mathematics education, and to explore the presence of naturally occurring mathematics in their national, ethnic, and family activities and heritage.

What tipped the scales in this case was the heterogeneity of the cultural background of the MPT population enrolled in the course. The demographic situation in the Negev region in the area of Beer-Sheva where Kaye College is located drives it to accept students in approximately equal numbers of Negev Arab-Bedouin (hereinafter: "Bedouins") (48%) and Jewish (52%) of the total of about 1,400 students, which was the number of students matriculating in all of the various academic tracks of the College during the year our research took place. Let us explain about the college population.

First, let us focus on the Negev Arab-Bedouins. This is a unique Palestinian community of Arabic-speaking desert nomads of the Middle East (Encyclopædia Britannica, Inc., 2007) that has lived in the Negev for centuries. In the 19th century the Bedouins began a transition to permanent settlements, which continued throughout the years. The process intensified from 1948, when the State of Israel was established. According to Boteach (2005), approximately 160,000 Arab Bedouins have settled in the Negev region of Israel. Arabs in Israel make up about 22 percent of the total population, which was about 6,990,700 at the end of 2005. Arabs living in the Negev constitute about two percent of the total Arab population of the State of Israel (Israel Central Bureau of Statistics, 2006).

The Bedouin population can be divided into two groups: roughly half of them lives in approximately 45 localities unrecognized by the State of Israel, follow the traditional Arab-Bedouin way of life, and in most cases lives in tents and shacks. The other half of the Bedouin population is concentrated in seven government-planned towns: Rahat, Tel Sheva, Kseife, etc,
planned as urban centers. The majority of Bedouins work in agriculture, construction, and traditional lifestyle works, among other fields. While formal education has been a part of Jewish tradition for centuries, the nomadic Bedouins began sending their children to educational settings only in 1948 under the Compulsory Education Law. Since then, the Bedouin of Israel have succeeded in reducing illiteracy from 95% to 25% (Ben-David, 2001). Most of Bedouins are religious Muslims that follow the Islamic calendar and culture. The cultural gap between Bedouin minority versus Jewish majority created an identity crisis for the Bedouins, when "56% of them regard themselves as Israelis and 61% as Palestinians" (Abu-Saad, Yonah and Kaplan, 2000). During the last thirty years a recognizable "process of integrating the Bedouin into Israeli society", has taken "place on two levels – the formal, by government policy; and the informal, by changing relationships with Israeli society in general and Jewish society in particular" (Ben-David, 2001). Moreover, Negev Bedouins have been changing/moving "from controllers of the desert region" to "fringe dwellers of a growing, modernizing Beer-Sheva city region (Yiftachel 2004). This can be seen in the range of Bedouins employed as clerks, lawyers, doctors, and teachers (at the time in small numbers).

Kaye College is the favorite Bedouin choice for higher education. The main reason for this is that the B.Ed. degree received with their graduation ensures them a job in the Negev area, close to their places of residence. Bedouins attend separate school systems than Jews, schools where the spoken language is Arabic. These school systems need competent Bedouin teachers to address the continual shortage. The Bedouin population attending the College from the Negev settlements still follows the traditional ways of life and preserves the ancient customs and traditions, as well as follows the religious imperatives of the Islam. In the college, these Bedouin students come in contact with their Jewish peers to perform mutual tasks during courses, but on
other occasions they prefer their own company, where the language they use is Arabic, which is also their communication language with Arab lecturers. Otherwise, with the Jewish students, Jewish lecturers and the management of the college, the language they speak is Hebrew.

Let us focus now on religious and secular Jewish population. The words 'secular' and 'religious' for Jews refer to the type of culture or lifestyle. From this point of view, secular and religious Jews are two separate groups. The secular Jews accept the Bible in general as the central source of "Jewish collective memory", and celebrate the Jewish Festivals and traditional ceremonies such as Brit Milah (circumcision) and Bar-Mitzvah (ceremony of a Jewish boy who has reached the age of 13). Despite the distinctions between different Jewish religious groups, the faithful observing the classical rules of Judaism (mostly Orthodox) and those like Conservative and Reform Jews who are not obliged to adhere to the religious traditions as the Orthodox do, nevertheless share a common traditional knowledge of religious rituals, habits, and behaviors. These variations explain the differences in the traditional practical mathematical knowledge (calendar calculations, mathematical calculations required to observe the religious commandments, etc) possessed by the religious MPT and their secular Jewish peers when they enter the college.

In some tracks the Bedouin and Jewish sectors in Kaye College learn separately; for example: MPT of Early Childhood Education - Jewish sector learn in Hebrew, and for the Bedouin sector in Arabic. However, the language used in the Math Education Department, including for the The History of Mathematics, is Hebrew for all. Kaye College contributes to educational activities in the Negev region and conducts many projects in both the Jewish and the Bedouin sectors, such as the project in which several Bedouin teachers are teaching Arabic in the Jewish school. In several tracks and departments, computer forums and social activities, co-
existence discussions take place, discussions which are in keeping with College policy. The
History of Mathematics course is the only course of the Math Department in which intercultural
conversation has been one of the pillars of the course syllabus since 1998. One of the course's
goals is to direct the students to think cross-culturally in accordance with the integrated tone of
Kaye College (Katsap, 2002).

**Research in the course**

The research that has accompanied the implementation of EP and was two-semesters in
length, dealt with examining possible effects likely to accrue for mathematics teachers' education
from learning and experiencing Ethnomathematics. As value added, the research attempted to
discover the nature of the social-cultural-mathematical discussion that takes place in the college
classroom during EP. 19 MPT (10 Jewish and 9 Bedouins) took active part in the EP by
preparing and presenting study units combining ethnomathematical contents in school math.
These students constituted the research population.

The research was carried out by qualitative study, collection and analysis of data
following the method of Grounded Theory (Glaser & Strauss, 1967).

The objectives of the research were:

1. To observe the educational process undergone by MPT

2. To examine the effects of EP on MPT by documenting:

   1) MPTs' attitudes toward interlacing of ethnomathematical content in school
      mathematics;

   2) MPT assessment of the EP;

   3) The feelings of MPT affected by exposure to ethnomathematics in the
      culture of the their own people and the surroundings in which they live;
4) The influence of learning environment during EP on the mathematical-social-cultural dialogue conducted in the college classroom;

5) The values emerging in the educational process of the MPT during EP.

The Grounded Theory (GT) research process was guided by the framework formed by two research models. The first model (proposed by Dick, 2005) is characterized by an algorithm built from six stages executed in the course of the research: 1) data collection; 2) note-taking; 3) coding; 4) memoing; 5) sorting; and 6) writing the report. The second model (proposed by Glasser & Strauss, 1967; Strauss & Corbin, 1998) supplements the first model through its focus and the great detail this model has with respect to the process of encoding. The combination of the two models enables optimum implementation of the GT method for construction of a theory regarding the phenomenon under investigation. Every sub-group of raw data passed through the first four stages according to Dick's model, with the primary coding being in the form of an open coding, close to the time of the gathering of the data. The method made it possible to receive memoranda that gradually built the theory on the basis of "the categories" and "the characteristics" that were coded, as well as the connection between them. It should be indicated also that in most cases, the names given to these categories were taken from the expressions of the teacher candidates themselves. The next phase of the data analysis was axial coding, described by Brown et al (2002) by means of four analytical processes: 1) continuing attribution of categories; 2) comparison between categories and data; 3) reducing the number of categories by determining their characteristics and fields; and 4) examination of the differences in the phenomena. With the continuation of this phase, the fifth and sixth stages were carried out, according to Dick (2005).
Integrating Ethnomathematics in school mathematics: contents and process

Two strand frameworks are identified with the field of Ethnomathematics: Geometry Patterns and Time Calculations were added to the syllabus of the course and served as mathematical content for the execution of the program. The four topics included in the first strand framework, "geometry patterns," were as follows:

- Learning Transformations: Researching Bedouin Dress Embroidery
- The Hemisphere Bowl Shape, The Circle, and The Jewish Kippah (small skullcap)
- Identifying Geometry Motifs in Bedouin Carpets
- Learning the Characteristics of the Equilateral Triangle and Investigating the Magen David (Star of David) Symbol of Jewish Culture

And in the second strand framework, "time calculations", such as the sequence of festivals, days, months and years, the following three topics were presented:

- Who Knows Eight? In the Jewish Festivals
- Mathematics in the Hebrew Calendar
- Mathematics in the Islamic Calendar

Diverse activities were utilized in order for the MPT to understand the complexity and application of these themes in each culture, with the goals being:

- Presenting the developmental, human, and cultural aspects of mathematics, and developing a comprehensive perception of mathematics through interlacing academic mathematics with cultural perspectives of mathematics;
- Understanding mathematical concepts and ideas through the prism of each other's culture - Jewish and Bedouin - and learning and solving mathematical problems as a part of mathematical knowledge developed over generations in both cultures;
Adapting the ethnomathematical content of mathematics to the scholastic content in teaching mathematics in elementary through high school.

Readers might wonder what the meaning of the topics titles were, and how they can connect academic mathematics and ethnomathematics. The topic titles were intended to set academic mathematics, on the one hand, and practical mathematics identified by each culture, Jewish and Bedouins, on the other, in minimum words, in language understood both by school pupils and by aspiring teachers from each respective culture. For the MPT in the course the above topics titles were very clear.

Henceforth, I will focus on three of the four topics that were presented on the theme of geometry patterns: a) Learning Transformations: Researching Bedouin Dress Embroidery; b) The Hemisphere Bowl Shape, The Circle, and The Jewish Kippah; and c) Geometric Motifs in Bedouin Carpets.

Learning Transformations: Researching Bedouin Dress Embroidery

The young Bedouin woman has learned fixed geometric patterns of embroidery from her mother. She has learned from her mother the requisite knowledge and skills for embroidery and has cultivated an attitude that demands she carry out the process with perfection. Moreover, she invests much thought in rearranging the patterns she has learned in order to make an original new creation. The color in the embroidery symbolizes the woman's status in society: for example, red embroidery symbolizes a fertile woman; in contrast, blue embroidery means a widow.

The group that presented this topic in the class showed that geometric patterns serve as a basis. These patterns appear in the Arabic Bedouin traditional dress embroidery identified with the composition that characterizes the dress in the Negev. At the time of the topic presentation class members are asked to learn about the tradition of embroiders, and to provide details about
how they managed to construct the composition requiring basic knowledge of squares, triangles, rhombs, and the most common forms in the Muslim world – "the symmetry: of half and quarter, of repeated repetition, diamond symmetry and symmetry of color" (Tal, 1990). One of the more interesting shapes appearing on the embroidery is the octagonal star, which, according to the aspiring teachers’ explanations, depicts a Bedouin settlement with the Sheikh's house in the middle (the point at the hub of the shape, see Figure 1).

![Figure 1: Octagonal star in the Bedouin embroideries](attachment:octagonal_star.jpg)

To summarize, it can be said that the background and the activities the aspiring teachers have learned worked well as a conclusion of the geometry topic quadrilaterals, and they were helpful for developing the skills needed for understanding different types of isometric transformations: reflection, rotation, translation, or glide reflection that appear in the mathematics curriculum of the school children. Following is an example of one of the activities implemented in the college class (includes pictures of embroideries to be used, Activity Sheet 1):
Activity Sheet 1

Learning Transformations: Researching Bedouin Dress Embroidery

Activity Instructions

1. Imagine and describe in your own words the feeling of the embroidering Bedouin woman.
2. Observe the designs appearing above, write a list of the symmetry types identifiable in the embroideries and sketch figures having the translational symmetry.
3. Find basic geometric patterns in the embroideries and copy them to the page. Form a new composition for the embroidery by using these figures.
4. Describe your feeling during the creation of embroidery "of your own".

The Hemisphere Bowl Shape, The Circle, and The Jewish Kippah

From 1500 CE onwards the tradition of covering the head became an obligatory practice for Jewish religious men (Raskin, 1990). Over the past three centuries, in some European
communities, the hat has evolved into the smaller skullcap – **kippah** in Hebrew. In Israel, wearing a kippah also has a social significance. Religious men don't appear in public without a kippah or some other headdress, in contrast to Secular men who wear a kippah while attending religious ceremonies. Moreover, the color of the kippah indicates affiliation with a particular religious group or stream. Thus, a black cloth kippah indicates that the wearer belongs to the orthodox stream of Judaism; other believers wear colored kippah. The kippah differs from any other hat, but it is a common convention that it must cover the crown of head, it is a three-dimensional, and it ends in a size that equal to a circle circumference of the head.

A group of MPT presented the subject of the kippah in the college class. They discussed the kippah wearer’s identification with socio-cultural and religious affiliations and also the symmetry in the kippah's decoration, size, and shape. The class members were required to identify types of symmetry and to examine the circle that is the basis of the kippah. Note that the circle has fascinated the Jews since biblical times, with verses depicting the calculation of $\pi$ appearing in the Bible (see more about math in Jewish writings in Lesser (2006)). One of the activities required students to gather information a kippah maker would need in order to make a kippah for a class member. To determine the circumference of the kippah, they measured around the student’s head as far down his head as he would wear the kippah. Suppose the kippah maker is using a knitting process similar to coiling a clay rope in making a pot, but from the lip to the bottom. To begin, the kippah maker needs to know the radius of the kippah at its base. Having the necessary circumference, the students calculated the radius by manipulating the formula $C = 2\pi r$. Actual kippah makers start knitting with a flat circular form, perhaps a cloth disc of the required circumference. As the process moves along, the kippah maker uses fewer and fewer “eyes” for each successive level of the kippah until he finally arrives at the pinnacle, having

Another activity examined different shapes of the kippah, inter alia, the Bukharan kippah that was influenced by the dominant Muslim culture where they lived in Uzbekistan. This kippah looks like a cylinder open at the base, flat on top like a round cake pan, not very deep, and sized with a circumference to fit one’s head, a shape that contrasts with the more common hemisphere, like a round bowl. An example of these kippah shapes and the corresponding activities implemented in the college class appears on Activity Sheet 2.
Activity Sheet 2

The Hemisphere Bowl Shape, Circle, and The Jewish Kippah

Bucharian Kippah  Crochet Kippah  A man wear a Kippah

Activity Instructions

1. Think about the Kippah-wearing custom in Jewish Culture and the events in which the males in the family wore Kippahs.
2. Observe the three Kippah's above and describe the shape, size and the decoration of the Kippah. Write a list of the symmetry types identifiable in each Kippah.
3. Select the basic geometric shapes from one or more Kippah's, copy them to the page and use them to make new decoration of a Kippah.
4. Describe your feeling while draw the decoration of a Kippah of "your own".
Identifying Geometric Motifs in Bedouin Carpets

Carpets serve as a supplement of Bedouins way of life, especially for those observing the traditional Arab-Bedouin community lifestyle and live in tents and shacks. A Bedouin tent is customarily divided into two sections by a woven carpets, one section reserved for the men and for the reception of most guests, and used as a gathering room for Bedouin men. The other section, in which the women cook and receive female guests, is the place of the women.

Both sections are padded with hand woven carpets to help guests stay warm during frosty seasons and to decorate their sitting places. The colors and geometric figures in the carpets are designated for special purposes; for example, geometric multi-color composition or red color background carpet is used only for exclusive persons in contrast with strips in white-leaden color background indicating that that carpet is for the family.

The group presenting the topic of "Identifying Geometric Motifs in Bedouin Carpets" made a systematic investigation of different carpets found in the area. They brought to the class stories (see below) from women elders in their families. Group members reported about instances of choosing figures for woven carpets and showed photographs of original carpets of their own people.

Among the activities the class members were asked to perform during the presentation of the topic "Identifying Geometric Motifs in Bedouin Carpets" was the completion of the left side of the carpet image below:
Presenters led their classmates to complete the symmetrical design in the image of the whole carpet by modeling the process, which the mother of one of the students elaborated, that Bedouin women customarily use. This woman had woven the carpet (see Activity Sheet 3, the carpet on the right), which was among the carpets that presenters had brought to class. To complete this classroom activity, class members made a drawing of vertical lines left of the midline, as shown in the image above. Next, the MPT measured the width of each part, starting at the right-hand edge of the carpet with the red swath and proceeding successively to the thin white strip, then to the maroon, and so on to the midline of the image. The most intricate element in the design, of course, is in the center and would seem to require careful measuring and counting of stitches, though the presenters did not state the details. In this activity, class members filled in the left half of the image, as a Bedouin woman would do in weaving a real carpet of this type, moving leftward from the midline all the way to the left edge, in essence symmetrically reflecting over the midline. And so by enacting this process on the carpet image, the MPT simulated the completion of a carpet in a way that Bedouin women use to weave actual carpets that feature reflection symmetry in method of creation and also in the design itself, as shown in Activity Sheet 3. In actual practice of weaving a carpet, of course, the Bedouin women move freely and smoothly from the right to left. The design they have in mind guides their work. Once the right-hand half of the carpet is complete, the midline of the carpet acts as a line of
reflection as they proceed to weave toward the left-hand edge, matching each new section in size and color to the corresponding one they have already woven to the right of the midline.
Activity Sheet 3

Identifying Geometric Motifs in Bedouin Carpets

Two original carpets from the house of one member of the group that prepared the study unit

Activity Instructions

1. Observe the designs appearing above and write a list of the symmetry types identifiable in the carpets.
2. Find basic geometric figures on the carpets, copy them to the page and explore their features.
3. What kinds of triangles have been woven on the carpets?
4. Describe your feeling toward the Bedouins carpets.
The conversation conducted inside the college class about the geometric figures and the kinds of geometric transformations appeared in the carpets built a bridge between academic mathematics and outside classroom life, and it permitted mathematical-socio-cultural dialogue, thus facilitating better understanding of several transformations. Photos of two carpets shown in the class and the questions the class members were asked during the activity appear in Activity Sheet 3.

After all presentations, the MPT used ethnomathematical ideas from dress embroidery, knitting the kippah, woven carpets, etc, to design new activities for school math in the fields of Geometry, Transformations, and Measurement.

**Intercultural and multiplex educational dialogue in the course**

One of the salient characteristics of the educational process that took place in the course was an intercultural dialogue. In any other study framework in Israel, it would have been unusual to emphasize national and religious differences in the class. Let us describe a case.

In a lesson on the concept of time presented by a Jewish group, the discussion ranged from the philosophical understanding of the concept to mathematical insight. When a Bedouin student presented his own position on the matter without referring to his culture’s attitude, one of the presenters immediately said, “Why are you presenting this example? It would be better if you told us how you view time in the desert. What does your sheikh (literally, "elder") think about the essence of time?” (See below regarding mathematics and Islamic calendar). In any other class, a political argument could have easily ensued. However, in this case, the Bedouin prospective teacher began to relate how Bedouin religious leaders saw the concept of time. Everyone listened with interest, as moments before the class had discussed the Western attitude
of "time is money." Two opinions from the two respective cultures represented their emotional perceptions of time and their cognitive mathematical perceptions of time.

Dialogue opened up the possibility of a renewed perception or deepening of understanding of the mathematical concepts – not only from the purely rational side but also from the affective side, with the latter being based on feelings rooted in the traditions and cultures of the MPTs. The clash of perspectives common outside the classroom was introduced into the classroom during the study and discussion of the mathematical and scientific notion of time. Incorporating an ethnomathematical perspective made this interchange possible, indeed encouraged it, and humanized and augmented the study academic mathematics.

**How the learning and teaching environment affects the ethnomathematics experience of the MPT**

The educational process in the course was comprised from three phases: the first phase was characterized by groups each comprising two or three MPT choosing a topic of mutual interest, collecting relevant material, and studying and learning collaboratively through investigation of the topic. The second phase was typified by students’ preparations for their presentations. So in their group work they attended to pedagogical considerations for activities their class mates would do, activities school students would do, and for subsequent class discussion and reflection with their peers. And the third phase, classroom presentation of the topic, was, according to all MPT, an "exciting learning experience."

The first stage was each group’s search for sources of information for their topic. From what the MPT indicated, in this search they turned to the following resources: the Internet Web literature, encyclopedias, books that focused on religious or cultural source materials, such as the Jewish Bible (Tanakh in Hebrew; Old Testament), the Talmud and other books of interpretation,
among the Jews, and the Koran and the Hadith among the Bedouin. In these sources of information the MPT found descriptions regarding handcrafts, such as knitting, weaving, embroidery and creative works, containing integrated patterns of certain cultural character that could be clearly identified. Regarding mathematics concepts and processes, they formed explanations regarding counting and ordering, such as calculations for calendar arrangement and fixing the dates on which the religious festivals fall. The MPT claimed that the information sources they prized most highly in the authentic material were in fact the human sources: 1) an authority in the particular field; 2) a religious personality; 3) an expert or professional; or 4) a family member (mother, father or family elders). All Jewish and Bedouin sectors participants emphasized that most of the people to whom they turned did not know at all how to express the connection between mathematics and their work, but they had mastered the authentic field itself. These reports are reminiscent of a marvelous story that Fasheh (2002), who holds a Ph. D. in mathematics, told about his "illiterate mother" who was a seamstress, and how she encountered math in her daily work. He was amazed to discover that his "mother was illiterate in relation to his type of knowledge [i. e., academic mathematics], but he was illiterate in terms of her type of understanding and knowledge [i. e., ethnomathematics]."

Some of the statements made by the MPT regarding the sources of information are cited below. The first story relates to an information source, "Mathematics in the Islamic Calendar", by a Bedouin group of MPT. Members of this group became interested in the hour-by-hour schedule of the daily prayers that exists in the Bedouin tradition, and for this reason they went to the Bedouin settlement of Kseife (located in Beer Sheva area) in order to meet with an "information source." The story is presented below:
“I spoke with one of the elders of Kseife. I asked him to tell me how they know to set the
hours of prayer without using a clock. This was his answer:”

We pray five prayers a day: morning, noontime, afternoon, evening
and night, and the time for prayer is determined by the sun and the
moon. Morning prayer begins when the dawn rises and a red
horizon appears in the sky. Noontime prayer arrives when a man's
shadow is equal to his height. The time for afternoon prayer comes
when a man's shadow is longer than his height. Evening prayer
arrives when both the sky and the sun look red in the west. And
finally, the time for nighttime prayer comes when the red color in
the sky can no longer be seen.

Here is another story told by a member of the group that studied Identifying Geometry
Motifs in Bedouin Carpets:

My group chose to present a study unit on 'Geometric Motifs in
Bedouin Carpets.' I remembered that my home has carpets with
different geometric forms. Then I went to my mother and to the
elderly women in the family, and I asked them about the forms in
the carpets. They responded with surprise bordering on refusal:
What? You're interested in women's handicrafts? I explained to
them that this was an assignment I took upon myself as part of the
preparation of my paper for college, and that the subject of forms
is connected to geometry, an area that I am studying in the
framework of my degree studies in mathematics teaching. Their
immediate reaction was that I had come to mock their work, and
only when I reassured them and told about the geometric forms
appearing in the carpets that they had made, such as triangles and
squares, that they complied with my request, and began to tell me
how the work of carpet-weaving is done. In the end, they added
that they had never thought that what they were doing was
connected to mathematics.

Both stories share being collected from an "in vivo" source. An ethnomathematical
perspective enabled the MPT to identify and study the mathematics intrinsic in each of the two
accounts, just as Fasheh’s ethnomathematical perspective, acquired after his formal doctoral
studies in mathematics, enabled him to discern the active geometric knowledge his mother
applied in making clothing from scraps of cloth. Tribe elders told the teachers that most stories about Bedouin existence, tradition, and culture are passed orally from generation to generation by father to son and from mother to daughter.

Contrary to their Bedouin colleagues, the Jewish MPT, secular and religious alike, encountered an abundance of written material in the Jewish sources, providing very precise interpretations of how to conduct mathematical calculations to ensure that those coming to fulfill the mitzvahs (Jewish religious laws) shall indeed meet the theoretical and practical demands of the task. The texts described, explained, and directed the actions by using components linked to mathematical knowledge in the culture, religion, and tradition of the Jewish people. The information was revealed, as mentioned above, in books which provided a lot of information on the way days are counted to calculate the date of a certain holiday, how calendars are constructed, measurements taken, or Jewish symbols are built: Hanukkah lamp, Star of David, etc. They too, just like their Bedouin colleagues, did not miss the opportunity to obtain the information from a religious authority, which usually was, quite naturally, the local rabbi:

I began to prepare the work on the subject of "Mathematics in the Hebrew Calendar" by collecting information from books, and then asked my family members to check and add more information on the subject, and finally visited a rabbi (religious teacher or leader), asking him to explain and help me search for verses pertaining to our subject in the Torah.

Let us pass to the third stage because the second stage, which was preparation of the lesson array, was not characterized by any special or distinguishing components among the topics or the domains of mathematics. Implementing their projects in class was, indeed, an "exciting experience" in the opinion of the MPT; they described the process of class activation as the most significant for them, thanks to the fertile mathematical-social-cultural discussion that
developed during the lesson. They have also mentioned that during the first stages of investigating their chosen subject, they anticipated that the 'simplicity' characterizing the practical mathematics in the studied topics would fail to elicit much respect from their classmates for the mathematics inherent in the tasks associated with their topics. However, the questions asked during the following class activities and discussion indicated that ethnomathematics content did, in fact, arouse curiosity and create enrichment.

One Bedouin MPT expressed appreciation and was impressed by the positive feedback her group received from Jewish MPT in the college class after the presentation:

I was very happy during the presentation… because the material was linked to our culture, and everyone is proud when other people learn and become acquainted with their own culture… I've noticed that during the presentation of the study unit that the participating teachers showed considerable interest and desire to learn about our culture… Our group prepared many examples of embroidery and in one activity Jewish teachers were asked to describe orally the transformation types revealed by this example. At the end of the lesson the teachers described their sensations during this task. The satisfaction they've felt was evident.

The words in “our culture" bore two meanings. One, creation of linkage between the MPT's culture and mathematical concepts belonging to academic mathematics, and second, a sense of satisfaction with teaching math by using examples relaying the story of the MPT’s own people. It can be said that the ethnomathematical experience greatly contributed to the sensation that ethnomathematics can serve the teachers for modeling, which cannot be provided by books written in the language and symbolism of formal mathematics and usually associated with mathematics content courses.

The profile of educational process undergone by the MPT

Essential components of the MPT’s ethnomathematical education were examined through 5 categories:
1. The attitudes concerning humanist-social and cultural aspects of ethnomathematics;

2. The potential utilizations of ethnomathematical examples;

3. A feeling aroused by exposure to mathematics in the culture of their own people and surroundings in which they live;

4. The contribution of EP experience to intentions to promote mathematical ideas in cultural perspective;

5. The values ethnomathematics teachers' education calls forth.

Concerning the first two categories, about 90% of the MPT agreed on the importance of linking the math teacher's preparation with cultural perspectives of mathematics and with the benefit in integrating "ethnomathematical" subjects in Math teaching. A typical statement supporting this inference is the following:

Participation in the program helped me to understand that mathematics is in my roots and culture, that it served and still serves my people in preservation of a time-old tradition… Mathematics has an extra value… Ethnomathematics answers exactly the questions asked by so many students: "why should we learn mathematics?" This area offers a ladder we can use to climb from mathematics in our environment to the universal math… I see this as the main importance of ethnomathematical education.

The citation above and similar expressions point toward MPT's appreciation of ethnomathematical education and to their comprehension that mathematics rooted in the culture and environment in which they live and teach need not be detached from the academic mathematics curriculum they are expected to implement with children.

Ethnomathematical education emphasizes the connection between mathematics and other areas. The MPT acquire knowledge in interdisciplinary teaching when they appeal to history,
sacred texts, and holy books; geography, and other disciplines; and so on. These connections create an "uninterrupted continuum" in their general knowledge. Answers supplied by the MPT were used to compile a list of useful characteristics provided by integrating ethnomathematical contents in the school mathematics intended for teaching. The most popular characteristics, those mentioned in various forms by more than 80% of the MPT, were these four: a) rich learning of the lesson's topic, extending the pupils' "world of knowledge"; b) changing the attitude towards mathematics, which is not only equations and laws, but also a part of national, ethnic, and family culture; c) creation of inter-disciplinary integration; and d) breaking the routine frames of the Math lesson. These data show that the MPT assigned most of the benefits to the fact that integration of ethnomathematics content in school mathematics can enhance the Math lessons and quality of the pupils' learning.

It should be noted that the explicit use of the term "ethnomathematics" in the reactions of the MPT was scarce. The most common alternative for this term was "mathematics in people's culture" or "mathematics in daily life". Likewise, the importance of ethnomathematics was always appraised for its contribution or potential to help cope with the difficulties in learning academic mathematics.

Concerning category three, the feeling aroused by learning mathematics in the context of their own cultures, the Jewish and Muslim MPT were delighted with the experience in mathematics in their own cultures, which utilized practical mathematics present for many generations. The level of marveling expressed by the MPT stemmed from the fact that the acquaintance with their own culture occurred at the "junction" named mathematics. The application of the word "our" was motivated by pride, and the appreciation showed by one
culture in the class towards the practical mathematics of the other. The MPT reactions reflected three major emotions:

**Surprise** - from the ability to arouse interest through Ethnomathematics;

**Enjoyment** - learning and teaching ethnomathematics in the college classroom;

**Pride** - representing their own people and culture.

Representing the fourth category, the contribution of experience with ethnomathematics during the EP to the MPT’s intentions to promote mathematical ideas in cultural perspective was expressed in the course of the self-learning, and by observing the presentation of the other study units in the class. Thus, for example, the MPT discovered the mathematical knowledge required to achieve a sophisticated use of symmetry in carpet weaving.

Through their experiences with ethnomathematics and academic mathematics, the MPT showed that they refined and extended their understanding of mathematics content and pedagogy. For example, before the EP, the MPT understood symmetry as an abstract math concept, and they were much less likely to make connections between the abstract embodiments and everyday appearances of symmetry, but after their experiences with mathematics embedded in cultural context, they refined and extended their comprehension of this concept through actions of their families and elders: symmetry in evidence in Bedouin women’s dress embroidery, their carpet weaving, and also in the designing and making of Jewish kippahs. Every case presented sophisticated application of principles of symmetry, not in the abstract, but in the precise enactments of the processes to create the intended products. These processes are ways of knowing and understanding in action, not in the abstract. Anticipating their own future students, the MPT also thought that children in schools could encounter these concepts through
their own actions and their use of their own language to describe the processes in which they would be engaged.

Another example is knowledge of the rules for compilation of annual calendars in both cultures, Jewish and Bedouin. The extant facts pertaining to mathematical knowledge hidden, so to speak, in the life experiences of the people, spurred the desire of the MPT to transfer to their future pupils the message that Math "is here", all around them, part of their culture. The MPT expressed their hope that the same feeling of enthusiasm that they, themselves, experienced would also be experienced by their future pupils when exposed to mathematics in their culture and in the culture of others. Cultural discourse complemented the mathematical discourse and filled the classroom with a lot of friendliness, mutual respect, and marveling at the ethnomathematics inherent in both cultures.

Table 1: Distribution of values concerning Teachers' Enthnomathematical Education

<table>
<thead>
<tr>
<th>Mathematical-Studied Values</th>
<th>Mathematical-Pedagogic Values</th>
<th>Social-Cultural Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Discovering mathematical concepts in symbols characterizing the teacher's own culture;</td>
<td>a) The teacher as indicator of ethnomathematics' ability to generate pupils' Interest;</td>
<td>a) Contribution to the teacher's self-confidence and openness;</td>
</tr>
<tr>
<td>b) Refining and extending prospective teachers’ understanding of mathematical concepts and their own teaching;</td>
<td>b) Improving the teacher's ability to conduct discussion with the class due to the didactis provided by the field of ethnomathematics;</td>
<td>b) Strengthening the bond between the teacher and the legacy of his people;</td>
</tr>
<tr>
<td>c) Holistic viewing of the mathematical concepts and mathematics around us;</td>
<td>c) Fostering teacher's creativity by developing new teaching materials;</td>
<td>c) Extending the knowledge about tradition and culture of the teacher's own people;</td>
</tr>
<tr>
<td>d) Acquaintance with popular culture traditions and customs through studying of mathematical concepts.</td>
<td>d) Acquiring know-how for adjustment and integration of ethnomathematical subjects in school mathematical contents.</td>
<td>d) Creation of a social climate of mutual respect originating in exposure to mathematical knowledge existing in both cultures: Jewish and Arabic.</td>
</tr>
</tbody>
</table>
Original quotations of the MPT served to determine values formed during the educational process that they experienced (fifth category). The model proposed by Bishop (1996), which divides values into three groups - mathematical, mathematical-pedagogical, and general educational - was adjusted to the array of values extracted from the findings and grouped accordingly. MPT expressed these values (see Table 1 above) with respect to their experiences in the process of education for integration of ethnomathematical content and school mathematics.

In general we can say that the findings of the research point toward two trends: the first one is characterized by 'systemic betterment' by: a) improvement in teaching techniques; b) enriching the program of MPT education; c) creation of a college classroom environment conducive to integrating ethnomathematical and multicultural features with mathematics content and pedagogy, as one MPT puts it:

> For the first time the mathematical concepts were taught using symbols and attributes characteristic of our culture, holidays and calendar - this was an opportunity to feel proud of mathematics present in our culture." (See Table 1: Mathematical-Studied Values, a).

The second trend is revealed in improvement of qualities of the MPT by the achievement of the following goals: a) improvement of learning and teaching skills, boosting their interest as learners of mathematics, and deepening their own mathematical content knowledge; b) broadening of the MPT general knowledge, c) creating a linkage between the MPT with his people and their roots. Quoting one of the MPT:

> A subject that I did not previously like, such as the theory of different symmetries, I saw suddenly in a new way in this course, after it was connected to the culture of my people. It was easy to understand and I now like it, and therefore I definitely think that the process of exposing the teacher to the cultural aspects of the mathematical ideas is one that contributes to the training of the teacher." (See Table 1: Social-Cultural Values, d).
Conclusion

The status of the teacher in the intersection between society and pedagogy is an issue for discussion with respect to mathematics teacher education in general and with respect to Ethnomathematics as a constituent strand in mathematics teachers teacher education, in particular. In an effort to embrace and celebrate multicultural phenomena in the mathematics curriculum process, the Ethnomathematical Program was designed as a MPT professional development project integrating ethnomathematics, academic mathematics for aspiring teachers, and explorations into MPT’s own cultures. Orey & Rosa (2006) encouraged the initiative and claimed that "ethnomathematics may come to exert some influence on mathematics education if the researchers would develop rigorous activities that are linked to standards and curriculum that make mathematics a living subject for both teachers and students." Focusing on ethnomathematics as a part of the History of Mathematics course permitted the MPT in the course to grasp the history of practical mathematics in the culture of their own people and to conceive the perception of mathematical concepts as a mix of two significances: one meaning stemming from their abstract interpretation in academic mathematics that is preeminent in western culture, and the second, stemming from comprehension of practical math concepts in a culture, rooted for both Jews and Arabs in the East. This perception came into sight during class debates, discussions, and sharing of projects: that there is some common religious knowledge (e.g., import of number seven in passages in the Torah and in the Koran) and common ritual (e.g., Jewish and Muslim religious women must cover their heads) in both lines of the descendants of Abraham.

However, the big take-away point is that just an exposure to mathematics rooted in the socio-cultural setting of their own people apparently would not have altered the standpoints of
the class members toward the way in which they perceived mathematical concepts. More than that was required. In particular, the personal inquiries about the concepts in situations where mathematical practice actually occurs, was necessary. It was, in fact, the MPT’s construction of the study units and presentation therein of the mathematical concepts, previously encountered by MPT strictly in the abstract, that stimulated these aspiring teachers to plan to integrate ethnomathematics into their mathematics teaching in their future school assignments. The Ethnomathematics Program, based on modeling and the persuasion of practical examples, that apparently was instrumental to the outcome.

In a similar vein, Shirley (2006) noted such adaptation and attributed the outcome in his example to the class discussion of these issues. As with Shirley’s (2006) work, this research indicated that discussions became important to teachers and motivated their further pursuit and understanding of mathematics content and pedagogy. Indeed, the way these MPT encountered culturally embedded mathematics created a learning environment that encouraged and nurtured interdisciplinary contexts and personal connections with one another and to their lives outside the classroom walls. This approach cultivated an acceptance that academic mathematics is only one side of the mathematics coin, the flip side being the one on which mathematics shows itself as a humanistic and socially embedded discipline. This development suggests that responsibility on the shoulders of professional educators lies in caring for creation of a learning atmosphere where the two facets of mathematics shall walk side by side. Mathematicians and mathematics teacher educators must accept, as priority, the pursuit of a civilization with dignity for all, and look into the relations between two universals: 1). mathematics as a universal mode of thought and 2). survival with dignity as the most universal problem facing mankind (D’Ambrosio, 2006 and 2007). Incorporating Ethnomathematics into mathematics teacher education would seem to
advance this goal, given the personal respect and appreciation for one another and their respective heritages that developed across the cultural boundary between the two often highly contentious groups of which the MPT are members. We recommend future research to investigate transfer in the schools by MPT who have experienced the Ethnomathematics Program.

Our mission as educators is to lead the MPT’s experiences so that they are likely to link classroom encounters with mathematics content and pedagogy, on the one hand, to ethnomathematical activities, on the other, that have definite affective and cognitive meaning for the learners in their national, ethnic, or family cultural realms. This research has demonstrated that Ethnomathematics in mathematics teacher education can influence the MPT’s professional development so that the MPT incorporate humanistic and social aspects of mathematics as they move from the role of teacher education students to that of teachers of children. Ethnomathematics activities were apparently responsible for creating a classroom climate in which appreciation and respect for different cultures and traditions emerged and flourished. One would hope that such inclusion of Ethnomathematics in teacher education will advance the ideals of peace and a better world.

References


